XVI. *Researches in Physical Astronomy.* By John William Lubbock, *Esq. V.P. and Treas. R.S.*

Read June 9, 1831.

I PROPOSE in this paper to extend the equations I have already given for determining the planetary inequalities, as far as the terms depending on the squares and products of the eccentricities, to the terms depending on the cubes of the eccentricities and quantities of that order, which is done very easily by a Table similar to Table II. in my Lunar Theory; and particularly to the determination of the great inequality of Jupiter, or at least such part of it as depends on the first power of the disturbing force. That part which depends on the square of the disturbing force may I think be most easily calculated by the methods given in my Lunar Theory; but not without great care and attention can accurate numerical results be expected. I have however given the analytical form of the coefficients of the arguments in the development of R, upon which that inequality principally depends.

It is I think particularly convenient to designate the arguments of the planetary disturbances by indices. The system of indices adopted in this paper is given as appearing better adapted for the purpose than that used in my former paper on the Planetary Theory; but it is not advisable to make use of the same indices in this as in the Lunar Theory.

I have also given analytical expressions for the development of R to the terms multiplied by the squares and products of the eccentricities inclusive, and for the terms in $r\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)$ multiplied by the first power of the eccentricities, which are I believe the simplest that can be proposed.

The following are the arguments which occur in the Planetary Theory.

Column 1 contains the index.

—— 2 contains the index of the argument, which is symmetrical.

—— 3 contains the index used Phil. Trans. Part II. 1830, p. 349.

0	..	..	0	104	..	39	$4t+x-z=5nt-5n_{,}t-\varpi+\varpi_{,}$
1	..	..	$t=nt-n_{,}t$	110	50	57	$2z=2n_{,}t-2\varpi_{,}$
2	..	..	$2t=2nt-2n_{,}t$	111	61	63	$t-2z=nt-3n_{,}t+2\varpi_{,}$
3	..	..	$3t=3nt-3n_{,}t$	112	62	64	$2t-2z=2nt-4n_{,}t+2\varpi_{,}$
4	..	..	$4t=4nt-4n_{,}t$	113	63	65	$3t-2z=3nt-5n_{,}t+2\varpi_{,}$
10	30	7	$x=nt-\varpi$	114	64	66	$4t-2z=4nt-6n_{,}t+2\varpi_{,}$
11	41	6	$t-x=-n_{,}t+\varpi$	121	51	58	$t+2z=nt+n_{,}t-2\varpi_{,}$
12	42	12	$2t-x=nt-2n_{,}t+\varpi$	122	52	59	$2t+2z=2nt-2\varpi_{,}$
13	43	13	$3t-x=2nt-3n_{,}t+\varpi$	123	52	60	$3t+2z=3nt-n_{,}t-2\varpi_{,}$
14	44	14	$4t-x=3nt-4n_{,}t+\varpi$	124	54	61	$4t+2z=4nt-2n_{,}t-2\varpi_{,}$
21	31	8	$t+x=2nt-n_{,}t-\varpi$	130	..	69	$2y_{,}=2n_{,}t-2\nu_{,}$
22	32	9	$2t+x=3nt-2n_{,}t-\varpi$	131	..	71	$t-2y=nt-3n_{,}t+2\nu_{,}$
23	33	10	$3t+x=4nt-3n_{,}t-\varpi$	132	..	73	$2t-2y=2nt-4n_{,}t+2\nu_{,}$
24	34	11	$4t+x=5nt-4n_{,}t-\varpi$	133	..	..	$3t-2y=3nt-5n_{,}t+2\nu_{,}$
30	10	15	$z=n_{,}t-\varpi_{,}$	134	..	..	$4t-2y=4nt-6n_{,}t+2\nu_{,}$
31	21	20	$t-z=nt-2n_{,}t+\varpi_{,}$	141	..	68	$t+2y=nt+n_{,}t-2\nu_{,}$
32	22	21	$2t-z=2nt-3n_{,}t+\varpi_{,}$	142	..	70	$2t+2y=2nt-2\nu_{,}$
33	23	22	$3t-z=3nt-4n_{,}t+\varpi_{,}$	143	..	72	$3t+2y=3nt-n_{,}t-2\nu_{,}$
34	24	23	$4t-z=4nt-5n_{,}t+\varpi_{,}$	144	..	..	$4t+2y=4nt-2n_{,}t-2\nu_{,}$
41	11	16	$t+z=nt-\varpi_{,}$	150	250	..	$3x=3nt-3\varpi$
42	12	17	$2t+z=2nt-n_{,}t-\varpi_{,}$	151	261	..	$t-3x=-2nt-n_{,}t+3\varpi$
43	13	18	$3t+z=3nt-2n_{,}t-\varpi_{,}$	152	262	..	$2t-3x=-nt-2n_{,}t+3\varpi$
44	14	19	$4t+z=4nt-3n_{,}t-\varpi_{,}$	153	263	..	$3t-3x=-3n_{,}t+3\varpi$
50	110	26	$2x=2nt-2\varpi$	154	264	..	$4t-3x=nt-4n_{,}t+3\varpi$
51	121	25	$t-2x=-nt-n_{,}t+2\varpi$	161	251	..	$t+3x=4nt-n_{,}t-3\varpi$
52	122	24	$2t-2x=-2n_{,}t+2\varpi$	162	252	..	$2t+3x=5nt-2n_{,}t-3\varpi$
53	123	32	$3t-2x=nt-3n_{,}t-2\varpi$	163	253	..	$3t+3x=6nt-3n_{,}t-3\varpi$
54	124	33	$4t-2x=2nt-4n_{,}t+2\varpi$	164	254	..	$4t+3x=7nt-4n_{,}t-3\varpi$
61	111	27	$t+2x=3nt-n_{,}t-2\varpi$	170	210	..	$2x+z=2nt+n_{,}t-2\varpi-\varpi_{,}$
62	112	28	$2t+2x=4nt-2n_{,}t-2\varpi$	171	221	..	$t-2x-z=-nt-2n_{,}t+2\varpi+\varpi_{,}$
63	113	29	$3t+2x=5nt-3n_{,}t-2\varpi$	172	222	..	$2t-2x-z=-3n_{,}t+2\varpi+\varpi_{,}$
64	114	30	$4t+2x=6nt-4n_{,}t-2\varpi$	173	223	..	$3t-2x-z=nt-4n_{,}t+2\varpi+\varpi_{,}$
70	..	47	$x+z=nt+n_{,}t-\varpi-\varpi_{,}$	174	224	..	$4t-2x-z=2nt-5n_{,}t+2\varpi+\varpi_{,}$
71	81	46	$t-x-z=-2n_{,}t+\varpi+\varpi_{,}$	181	211	..	$t+2x+z=3nt-2\varpi-\varpi_{,}$
72	82	53	$2t-x-z=nt-3n_{,}t+\varpi+\varpi_{,}$	182	212	..	$2t+2x+z=4nt-n_{,}t-2\varpi-\varpi_{,}$
73	83	54	$3t-x-z=2nt-4n_{,}t+\varpi+\varpi_{,}$	183	213	..	$3t+2x+z=5nt-2n_{,}t-2\varpi-\varpi_{,}$
74	84	55	$4t-x-z=3nt-5n_{,}t+\varpi+\varpi_{,}$	184	214	..	$4t+2x+z=6nt-3n_{,}t-2\varpi-\varpi_{,}$
81	71	48	$t+x+z=2nt-\varpi-\varpi_{,}$	190	230	..	$2x-z=2nt-n_{,}t-2\varpi+\varpi_{,}$
82	72	49	$2t+x+z=3nt-n_{,}t-\varpi-\varpi_{,}$	191	231	..	$t-2x+z=-nt+2\varpi-\varpi_{,}$
83	73	50	$3t+x+z=4nt-2n_{,}t-\varpi-\varpi_{,}$	192	232	..	$2t-2x+z-n_{,}t+2\varpi-\varpi_{,}$
84	74	51	$4t+x+z=5nt-3n_{,}t-\varpi-\varpi_{,}$	193	233	..	$3t-2x+z=nt-2n_{,}t+2\varpi-\varpi_{,}$
90	..	35	$x-z=nt-n_{,}t-\varpi+\varpi_{,}$	194	234	..	$4t-2x+z=2nt-3n_{,}t+2\varpi-\varpi_{,}$
91	..	41	$t-x+z=\varpi-\varpi_{,}$	201	241	..	$t+2x-z=3nt-2n_{,}t-2\varpi+\varpi_{,}$
92	..	42	$2t-x+z=nt-n_{,}t+\varpi-\varpi_{,}$	202	242	..	$2t+2x-z=4nt-3n_{,}t-2\varpi+\varpi_{,}$
93	..	43	$3t-x+z=2nt-2n_{,}t+\varpi+\varpi_{,}$	203	243	..	$3t+2x-z=5nt-4n_{,}t-2\varpi+\varpi_{,}$
94	..	44	$4t-x+z=3nt-3n_{,}t+\varpi+\varpi_{,}$	204	244	..	$4t+2x-z=6nt-5n_{,}t-2\varpi+\varpi_{,}$
101	..	36	$t+x-z=2nt-2n_{,}t-\varpi+\varpi_{,}$	210	170	..	$x+2z=nt+2n_{,}t-\varpi-2\varpi_{,}$
102	..	37	$2t+x-z=3nt-3n_{,}t-\varpi+\varpi_{,}$	211	222	..	$t-x-2z=-3n_{,}t+\varpi+2\varpi_{,}$
103	..	38	$3t+x-z=4nt-4n_{,}t-\varpi+\varpi_{,}$	212	223	..	$2t-x-2z=nt-4n_{,}t+\varpi+2\varpi_{,}$

213	224	$3t-x-2z=2nt-5n_{,}t+\varpi+2\varpi_{,}$	282	..	$2t+x+2y=3nt-\varpi-2\nu_{,}$
214	225	$4t-x-2z=3nt-6n_{,}t+\varpi+2\varpi_{,}$	283	..	$3t+x+2y=4nt-n_{,}t-\varpi-2\nu_{,}$
221	171	$t+x+2z=2nt+n_{,}t-\varpi-2\varpi_{,}$	284	..	$4t+x+2y=5nt-2n_{,}t-\varpi-2\nu_{,}$
222	172	$2t+x+2z=3nt-\varpi-2\varpi_{,}$	290	..	$x-2y=nt-2n_{,}t-\varpi+2\nu_{,}$
223	173	$3t+x+2z=4nt-n_{,}t-\varpi-2\varpi_{,}$	291	..	$t-x+2y=n_{,}t+\varpi-2\nu_{,}$
224	174	$4t+x+2z=5nt-n_{,}t-\varpi-2\varpi_{,}$	292	..	$2t-x+2y=nt+\varpi-2\nu_{,}$
230	190	$x-2z=nt-2n_{,}t-\varpi+2\varpi_{,}$	293	..	$3t-x+2y=2nt-n_{,}t+\varpi-2\nu_{,}$
231	191	$t-x+2z=n_{,}t+\varpi-2\varpi_{,}$	294	..	$4t-x+2y=3nt-2n_{,}t+\varpi-2\nu_{,}$
232	192	$2t-x+2z=nt+\varpi-2\varpi_{,}$	301	..	$t+x-2y=2nt-3n_{,}t-\varpi+2\nu_{,}$
233	193	$3t-x+2z=2nt-n_{,}t+\varpi-2\varpi_{,}$	302	..	$2t+x-2y=3nt-4n_{,}t-\varpi+2\nu_{,}$
234	194	$4t-x+2z=3nt-2n_{,}t+\varpi-2\varpi_{,}$	303	..	$3t+x-2y=4nt-5n_{,}t-\varpi+2\nu_{,}$
241	201	$t+x-2z=2nt-3n_{,}t-\varpi+2\varpi_{,}$	304	..	$4t+x-2y=5nt-6n_{,}t-\varpi+2\nu_{,}$
242	202	$2t+x-2z=3nt-4n_{,}t-\varpi+2\varpi_{,}$	310	..	$z+2y=3n_{,}t-\varpi_{,}-2\nu_{,}$
243	203	$3t+x-2z=4nt-5n_{,}t-\varpi+2\varpi_{,}$	311	..	$t-z-2y=nt-4n_{,}t+\varpi_{,}+2\nu_{,}$
244	204	$4t+x-2z=5nt-6n_{,}t-\varpi+2\varpi_{,}$	312	..	$2t-z-2y=2nt-5n_{,}t+\varpi_{,}+2\nu_{,}$
250	150	$3z=3n_{,}t-3\varpi_{,}$	313	..	$3t-z-2y=3nt-6n_{,}t+\varpi_{,}+2\nu_{,}$
251	161	$t-3z=nt-4n_{,}t+3\varpi_{,}$	314	..	$4t-z-2y=4n_{,}t-7n_{,}t+\varpi_{,}+2\nu_{,}$
252	162	$2t-3z=2nt-5n_{,}t+3\varpi_{,}$	321	..	$t+z+2y=nt+2n_{,}t-\varpi_{,}-2\nu_{,}$
253	163	$3t-3z=3nt-6n_{,}t+3\varpi_{,}$	322	..	$2t+z+2y=2nt+n_{,}t-\varpi_{,}-2\nu_{,}$
254	164	$4t-3z=4nt-7n_{,}t+3\varpi_{,}$	323	..	$3t+z+2y=3nt-\varpi_{,}-2\nu_{,}$
261	151	$t+3z=nt+2n_{,}t-3\varpi_{,}$	324	..	$4t+z+2y=4nt-n_{,}t-\varpi_{,}-2\nu_{,}$
262	152	$2t+3z=2n+n_{,}t-3\varpi_{,}$	330	..	$z-2y=-n_{,}t-\varpi_{,}+2\nu_{,}$
263	153	$3t+3z=3nt-3\varpi_{,}$	331	..	$t-z+2y=nt+\varpi_{,}-2\nu_{,}$
264	154	$4t+3z=4nt-n_{,}t-3\varpi_{,}$	332	..	$2t-z+2y=2nt-n_{,}t+\varpi_{,}-2\nu_{,}$
270	..	$x+2y=nt+2n_{,}t-\varpi-2\nu_{,}$	333	..	$3t-z+2y=3nt-2n_{,}t+\varpi_{,}+2\nu_{,}$
271	..	$t-x-2y=-3n_{,}t+\varpi+2\nu_{,}$	334	..	$4t-z+2y=4nt-3n_{,}t+\varpi_{,}-2\nu_{,}$
272	..	$2t-x-2y=nt-4n_{,}t+\varpi+2\nu_{,}$	341	..	$t+z-2y=nt-2n_{,}t-\varpi_{,}+2\nu_{,}$
273	..	$3t-x-2y=2nt-5n_{,}t+\varpi+2\nu_{,}$	342	..	$2t+z-2y=2nt-3n_{,}t-\varpi_{,}+2\nu_{,}$
274	..	$4t-x-2y=3nt-6n_{,}t+\varpi+2\nu_{,}$	343	..	$3t+z-2y=3nt-4n_{,}t-\varpi_{,}+2\nu_{,}$
281	..	$t+x+2y=2nt+n_{,}t-\varpi-2\nu_{,}$	344	..	$4t+z-2y=4nt-5n_{,}t-\varpi_{,}+2\nu_{,}$

TABLE I.

Showing the arguments which result from the combination of the arguments 10, 50 and 150 with the arguments in the first or left-hand column, by addition and subtraction.

	10	50	150			10	50	150			10	50	150	
1	21 11	61 51	161 151	1	51	11 151	....	....	51	102	202 32	....	....	102
2	22 12	62 52	162 152	2	52	12 152	....	....	52	103	203 33	....	....	103
3	23 13	63 53	163 153	3	53	13 153	....	....	53	104	204 34	....	....	104
4	24 14	64 54	164 154	4	54	14 154	....	....	54	110	210 −230	....	....	110
10	50 0	150 − 10	....	10	61	161 21	....	....	61	111	241 211	....	....	111
11	1 51	21 151	....	11	62	162 22	....	....	62	112	242 212	....	....	112
12	2 52	22 152	....	12	63	163 23	....	....	63	113	243 213	....	....	113
13	3 53	23 153	....	13	64	164 24	....	....	64	114	244 214	....	....	114
14	4 54	24 154	....	14	70	170 30	....	....	70	121	221 231	....	....	121
21	61 1	161 11	....	21	71	31 171	....	....	71	122	222 232	....	....	122
22	62 2	162 12	....	22	72	32 172	....	....	72	123	223 233	....	....	123
23	63 3	163 13	....	23	73	33 173	....	....	73	124	224 234	....	....	124
24	64 4	164 14	....	24	74	34 174	....	....	74	130	270 −290	....	....	130
30	70 − 90	170 −190	....	30	81	181 41	....	....	81	131	301 271	....	....	131
31	101 71	201 171	....	31	82	182 42	....	....	82	132	302 272	....	....	132
32	102 72	202 172	....	32	83	183 43	....	....	83	133	303 273	....	....	133
33	103 73	203 173	....	33	84	184 44	....	....	84	134	304 274	....	....	134
34	104 74	204 174	....	34	90	190 − 30	....	....	90	141	281 291	....	....	141
41	81 91	181 191	....	41	91	41 191	....	....	91	142	282 292	....	....	142
42	82 92	182 192	....	42	92	42 192	....	....	92	143	283 293	....	....	143
43	83 93	183 193	....	43	93	43 193	....	....	93	144	284 294	....	....	144
44	84 94	184 194	....	44	94	44 194	....	....	94					
50	150 10	....	....	50	101	201 31	....	....	101					

Table II.

Showing the arguments which, by their combination with the arguments 10, 50, and 150, by addition and subtraction, produce the arguments in the first or left-hand column.

	10	50	150			10	50	150			10	50	150	
1	11 21	....	....	1	43	93 83	....	....	43	90	...... – 30	....	....	90
2	12 22	....	....	2	44	94 84	....	....	44	91	 41	....	....	91
3	13 23	....	....	3	50	10	0	....	50	92	 42	....	....	92
4	14 24	....	....	4	51	 11	 1	....	51	93	 43	....	....	93
10	0 50	...... – 10	....	10	52	 12	 2	....	52	94	 44	....	....	94
11	51 1	 21	....	11	53	 13	 3	....	53	101	31	....	....	101
12	52 2	 22	....	12	54	 14	 4	....	54	102	32	....	....	102
13	53 3	 23	....	13	61	21	1	....	61	103	33	....	....	103
14	54 4	 24	....	14	62	22	2	....	62	104	34	....	....	104
21	1 61	11	....	21	63	23	3	....	63	150	50	10	0	150
22	2 62	12	....	22	64	24	4	....	64	151	 51	 11	 1	151
23	3 63	13	....	23	70	30	....	....	70	152	 52	 12	 2	152
24	4 64	14	....	24	71	 31	....	....	71	153	 53	 13	 3	153
30	70. – 90	....	....	30	72	 32	....	....	72	154	 54	 14	 4	154
31	71 101	....	....	31	73	 33	....	....	73	161	61	21	1	161
32	72 102	....	....	32	74	 34	....	....	74	162	62	22	2	162
33	73 103	....	....	33	81	41	....	....	81	163	63	23	3	163
34	74 104	....	....	34	82	42	....	....	82	164	64	24	4	164
41	91 81	....	....	41	83	43	....	....	83	170	70	30	....	170
42	92 82	....	....	42	84	44	....	....	84	171	 71	 31	....	171

TABLE II. (Continued.)

	10	50	150			10	50	150			10	50	150	
172	 72	 32	....	172	212	 112	....	....	212	272	 132	....	....	272
173	 73	 33	....	173	213	 113	....	....	213	273	 133	....	....	273
174	 74	 34	....	174	214	 114	....	....	214	274	 134	....	....	274
181	81	41	....	181	221	121	....	....	221	281	141	....	....	281
182	82	42	....	182	222	122	....	....	222	282	142	....	....	282
183	83	43	....	183	223	123	....	....	223	283	143	....	....	283
184	84	44	....	184	224	124	....	....	224	284	144	....	....	284
190	90	...... – 30	....	190	230	...... –110	....	....	230	290	...... –130	....	....	290
191	 91	 41	....	191	231	 121	....	....	231	291	 141	....	....	291
192	 92	 42	....	192	232	 122	....	....	232	292	 142	....	....	292
193	 93	 43	....	193	233	 123	....	....	233	293	 143	....	....	293
194	 94	 44	....	194	234	 124	....	....	234	294	 144	....	....	294
201	101	31	....	201	241	111	....	....	241	301	131	....	....	301
202	102	32	....	202	242	112	....	....	242	302	132	....	....	302
203	103	33	....	203	243	113	....	....	243	303	133	....	....	303
204	104	34	....	204	244	114	....	....	244	304	134	....	....	304
210	110	....	....	210	270	130	....	....	270					
211	 111	....	....	211	271	 131	....	....	271					

The following examples will show the use of the preceding Table, in forming the equations of condition which serve to determine the coefficients of the inequalities of the reciprocal of the radius vector and of the longitude.

$$-\frac{d^2 . r^3 \delta \frac{1}{r}}{d t^2} - \mu \delta . \frac{1}{r} + 2 \int d R + r \left(\frac{d R}{d r}\right) = 0$$

$$r^3 = a^3 \left\{1 + 3e^2\left(1 + \frac{e^2}{8}\right) - 3e\left(1 + \frac{3}{8}e^2\right)\cos(nt + \varepsilon - \varpi) + \frac{e^3}{8}\cos(3nt + 3\varepsilon - 3\varpi)\right.$$

$$\frac{(n - n_{\prime})^2}{n^2}\left\{(1 + 3e^2)r_1 - \frac{3e^2}{2}(r_{11} + r_{21})\right\} - r_1 + \frac{m_{\prime}}{a}q_1 = 0$$

$$\frac{4(n - n_{\prime})^2}{n^2}\left\{(1 + 3e^2)r_2 - \frac{3}{2}e^2(r_{12} + r_{22})\right\} - r_2 + \frac{m_{\prime}}{a}q_2 = 0$$

$$\frac{d\lambda}{dt} = \frac{h}{r^2} + \frac{2h}{r}\delta \cdot \frac{1}{r} - \frac{1}{r^2}\int \frac{dR}{d\lambda}dt$$

$$\frac{a^2}{r^2} = 1 + \frac{e^2}{2} + 2e\left(1 + \frac{3e^2}{8}\right)\cos(nt + e - \varpi) + \frac{5e^2}{2}\cos(2nt + 2\varepsilon - 2\varpi)$$

$$+ \frac{13}{4}e^3\cos(3nt + 3\varepsilon - 3\varpi)$$

$$\frac{a}{r} = 1 + e\left(1 - \frac{e^2}{8}\right)\cos(nt + \varepsilon - \varpi) + e^2\cos(2nt + 2\varepsilon - 2\varpi) + \frac{9}{8}e^3\cos(3nt + 3\varepsilon - 3\varpi)$$

$$\lambda = n\{1 + 2r_0\}t + \varepsilon$$

$$+\left\{2\left\{r_1 + \frac{e^2}{2}(r_{11} + r_{21})\right\}\right.$$

$$\left. - \frac{m_{\prime}}{\mu}\left\{\left(1 + \frac{e^2}{2}\right)\frac{anR_1}{(n - n_{\prime})} + \frac{e^2}{n_{\prime}}anR_{11} + \frac{e^2anR_{21}}{(2n - n_{\prime})}\right\}\right\}\frac{n}{(n - n_{\prime})}\sin(nt - n_{\prime}t + \varepsilon - \varepsilon_{\prime})$$

$$+\left\{2\left\{r_2 + \frac{e^2}{2}(r_{12} + r_{22})\right\}\right.$$

$$\left. - \frac{m_{\prime}}{\mu}\left\{\left(1 + \frac{e^2}{2}\right)\frac{anR_2}{(n - n_{\prime})} + \frac{2e^2anR_{12}}{(n - 2n_{\prime})} + \frac{2e^2anR_{22}}{(3n - 2n_{\prime})}\right\}\right\}\frac{n}{2(n - n_{\prime})}\sin(2nt - 2n_{\prime}t + \varepsilon - \varepsilon_{\prime})$$

In the same way, by means of the Table, all the other coefficients may be found.

The great inequality of Jupiter consists of the arguments 155, 174, 213, 273, and 312, the variable part of which is $2n - 5n_{\prime}$, and arises, as is well known, from the introduction of the square of this quantity, which is small, by successive integrations in the denominators of the coefficients of the sines in the expression for the longitude, of which the above named are the arguments.

The following are the equations which have reference to these arguments, and which may be found at once by Table II.

$$\frac{(2n - 5n_{\prime})^2}{n^2}\left\{r_{155} - \frac{3}{2}r_{54} + \frac{1}{16}r_4\right\} - r_{155} + \frac{m_{\prime}a}{\mu}q_{155} = 0$$

$$\frac{(2n - 5n_{\prime})^2}{n^2}\left\{r_{174} - \frac{3}{2}r_{74}\right\} - r_{174} + \frac{m_{\prime}a}{\mu}q_{174} = 0$$

$$\frac{(2n - 5n_{\prime})^2}{n^2}\left\{r_{213} - \frac{3}{2}r_{113}\right\} - r_{213} + \frac{m_{\prime}a}{\mu}q_{214} = 0$$

$$\frac{(2n-5n_{\prime})^2}{n^2}\left\{r_{273}-\frac{3}{2}r_{133}\right\}-r_{273}+\frac{m_{\prime}a}{\mu}q_{273}=0$$

$$\frac{(2n-5n_{\prime})^2}{n^2}\left\{r_{312}-r_{312}\right\}+\frac{m_{\prime}a}{\mu}q_{312}=0$$

$$\delta\lambda=\left\{2\left\{r_{155}+\frac{1}{2}\left(r_{55}+r_{15}+\frac{9}{8}r_4\right)\right\}\right.$$

$$\left.-\frac{m_{\prime}}{\mu}\left\{\frac{5na}{(2n-5n_{\prime})}R_{155}+\frac{5naR_{55}}{(3n-5n_{\prime})}+\frac{5.5naR_{15}}{4(3n-4n_{\prime})}+\frac{13.5naR_5}{8.5(n-n_{\prime})}\right\}\right\}\frac{ne^3}{(2n-5n_{\prime})}\sin(2nt-5n_{\prime}t+3\varpi)$$

$$+\left\{2\left\{r_{174}+\frac{1}{2}(r_{74}+r_{34})\right\}\right.$$

$$\left.-\frac{m_{\prime}}{\mu}\left\{\frac{4naR_{174}}{(2n-5n_{\prime})}+\frac{4naR_{74}}{(3n-5n_{\prime})}+\frac{5.4.naR_{34}}{4(4n-5n_{\prime})}\right\}\right\}\frac{ne^2e_{\prime}}{(2n-5n_{\prime})}\sin(2nt-5n_{\prime}t+2\varpi+\varpi_{\prime})$$

$$+\left\{2\left\{r_{213}+\frac{1}{2}r_{113}\right\}-\frac{m_{\prime}}{\mu}\left\{\frac{3naR_{213}}{(2n-5n_{\prime})}+\frac{3naR_{113}}{(3n-5n_{\prime})}\right\}\right\}\frac{nee_{\prime}^2}{(2n-5n_{\prime})}\sin(2nt-5n_{\prime}t+\varpi+2\varpi_{\prime})$$

$$+\left\{2\left\{r_{273}+\frac{1}{2}r_{133}\right\}-\frac{m_{\prime}}{\mu}\left\{\frac{3naR_{273}}{(2n-5n_{\prime})}+\frac{3naR_{133}}{(3n-5n_{\prime})}\right\}\right\}\frac{ne\sin^2\frac{\iota_{\prime}}{2}}{(2n-5n_{\prime})}\sin(2nt-5n_{\prime}t+\varpi+2\nu_{\prime})$$

$$+\left\{2r_{312}-\frac{2m_{\prime}naR_{312}}{\mu(2n-5n_{\prime})}\right\}\frac{ne_{\prime}\sin^2\frac{\iota_{\prime}}{2}}{(2n-5n_{\prime})}\sin(2nt-5n_{\prime}t+\varpi_{\prime}+2\nu_{\prime})$$

The quantities r_{55}, r_{74}, r_{113} and r_{133} have the quantity $2n-5n_{\prime}$ in the denominator, rejecting those quantities in the value of $\delta\lambda$ which have not $(2n-5n_{\prime})^2$ in the denominator.

$$r_{155}=-\frac{4m_{\prime}n^3aR_{155}e^3}{\mu(n-5n_{\prime})(3n-5n_{\prime})(2n-5n_{\prime})}$$

$$r_{174}=-\frac{4m_{\prime}n^3aR_{174}e^2e_{\prime}}{\mu(n-5n_{\prime})(3n-5n_{\prime})(2n-5n_{\prime})}$$

$$r_{213}=-\frac{4m_{\prime}n^3aR_{213}ee_{\prime}^2}{\mu(n-5n_{\prime})(3n-5n_{\prime})(2n-5n_{\prime})}$$

$$r_{273}=-\frac{4m_{\prime}n^3aR_{273}e\sin^2\frac{\iota_{\prime}}{2}}{\mu(n-5n_{\prime})(3n-5n_{\prime})(2n-5n_{\prime})}$$

$$r_{312}=-\frac{4m_{\prime}n^3aR_{312}e_{\prime}\sin^2\frac{\iota_{\prime}}{2}}{\mu(n-5n_{\prime})(3n-5n_{\prime})(2n-5n_{\prime})}$$

$$\delta\lambda=\left\{2r_{155}+r_{55}-\frac{5m_{\prime}naR_{155}}{\mu(2n-5n_{\prime})}\right\}\frac{ne^3}{(2n-5n_{\prime})}\sin(2nt-5n_{\prime}t+3\varpi)$$

$$+\left\{2r_{174}+r_{74}-\frac{4m_{\prime}naR_{174}}{\mu(2n-5n_{\prime})}\right\}\frac{ne^2e_{\prime}}{(2n-5n_{\prime})}\sin(2nt-5n_{\prime}t+2\varpi+\varpi_{\prime})$$

$$+\left\{2\,r_{213}+r_{113}-\frac{3\,m_{\prime}\,a\,n\,R_{213}}{\mu(2\,n-5\,n_{\prime})}\right\}\frac{n\,e\,e_{\prime}^{2}}{(2\,n-5\,n_{\prime})}\sin\left(2\,n\,t-5\,n_{\prime}t+\varpi+2\,\varpi_{\prime}\right)$$

$$+\left\{2\,r_{273}+r_{133}-\frac{3\,m_{\prime}\,a\,n\,R_{273}}{\mu(2\,n-5\,n_{\prime})}\right\}\frac{n\,e\sin^{2}\frac{\iota_{\prime}}{2}}{(2\,n-5\,n_{\prime})}\sin\left(2\,n\,t-5\,n_{\prime}t+\varpi+2\,\nu_{\prime}\right)$$

$$+\left\{2\,r_{312}-\frac{2\,m_{\prime}\,a\,n\,R_{312}}{\mu(2\,n-5\,n_{\prime})}\right\}\frac{n\,e_{\prime}\sin^{2}\frac{\iota_{\prime}}{2}}{(2\,n-5\,n_{\prime})}\sin\left(2\,n\,t-5\,n_{\prime}t+\varpi_{\prime}+2\,\nu_{\prime}\right)$$

The coefficients of the terms in the development of R multiplied by the cubes of the eccentricities, as regards the quantities b_5 and b_7, (they also contain the quantities b_3,) may be found by changing b_3 into b_5, in the terms in R multiplied by the eccentricities, and multiplying the result by

$$-\underset{[0]}{\frac{9}{8}\frac{(a^2e^2+a_{\prime}^2e_{\prime}^2)}{a_{\prime}^2}}+\underset{[50]}{\frac{3}{8}\frac{a^2}{a_{\prime}^2}e^2\cos 2\,x}-\underset{[1]}{\frac{3}{4}\frac{a}{a_{\prime}}\left(e^2+e_{\prime}^2+2\sin^2\frac{\iota_{\prime}}{2}\right)\cos t}+\underset{[61]}{\frac{9}{16}\frac{a}{a_{\prime}}e^2\cos(t+2\,x)}$$

$$-\underset{[111]}{\frac{9}{16}\frac{a}{a_{\prime}}e_{\prime}^2\cos(t-2\,z)}+\underset{[51]}{\frac{3}{16}\frac{a}{a_{\prime}}e^2\cos(t-2\,x)}+\underset{[121]}{\frac{3}{4}\frac{a}{a_{\prime}}e_{\prime}^2\cos(t+2\,z)}+\underset{[91]}{\frac{27}{8}\frac{a}{a_{\prime}}e\,e_{\prime}\cos(t-x+z)}$$

$$-\underset{[81]}{\frac{9}{8}\frac{a}{a_{\prime}}e\,e_{\prime}\cos(t+x+z)}-\underset{[71]}{\frac{9}{8}\frac{a}{a_{\prime}}e\,e_{\prime}\cos(t-x-z)}+\underset{[101]}{\frac{3}{8}\frac{a}{a_{\prime}}e\,e_{\prime}\cos(t+x-z)}$$

$$+\underset{[141]}{\frac{3}{2}\frac{a}{a_{\prime}}\sin^2\frac{\iota_{\prime}}{2}\cos(t+2\,y)}+\underset{[110]}{\frac{3}{8}e_{\prime}^2\cos 2\,z}$$

and changing b_5 into b_7, in the terms in R multiplied by the squares and products of the eccentricities, and multiplying the result by

$$-\frac{5}{6}\text{ and }-\underset{[10]}{\frac{2\,a^2}{a_{\prime}^2}e\cos x}+\underset{[11]}{\frac{3\,a}{a_{\prime}}e\cos(t-x)}+\underset{[41]}{\frac{3\,a}{a_{\prime}}e_{\prime}\cos(t+z)}-\underset{[21]}{\frac{a}{a_{\prime}}e\cos(t+x)}$$

$$-\underset{[31]}{\frac{a}{a_{\prime}}e_{\prime}\cos(t-z)}-\underset{[30]}{2\,e_{\prime}\cos z}$$

and changing b_3 into b_5 in the terms in R multiplied by the squares and products of the eccentricities, and multiplying the result by $-\frac{3}{4}$ and the same quantity.

Thus R_{155} results from the combination of the arguments

51 × 14, 50 × 15, 61 × 16, 10 × 55, and 11 × 54.

$$51\times 14\text{ gives }+\frac{3}{32}\frac{a}{a_{\prime}}\left\{\frac{3\,a}{4\,a_{\prime}^2}b_{5,3}-\frac{a^2}{2\,a_{\prime}^3}b_{5,4}-\frac{a}{4\,a_{\prime}^2}b_{5,5}\right\}$$

$$50 \times 15 \text{ gives } + \frac{3}{16}\frac{a^2}{a_i^2}\left\{\frac{3a}{4a_i^2}b_{5,4} - \frac{a^2}{2a_i^3}b_{5,5} - \frac{a}{4a_i^2}b_{5,6}\right\}$$

$$61 \times 16 \text{ gives } + \frac{9}{32}\frac{a}{a_i}\left\{\frac{3a}{4a_i^2}b_{5,5} - \frac{a^2}{2a_i^3}b_{5,6} - \frac{a}{4a_i^2}b_{5,7}\right\}$$

$$R_{55} = -\frac{a}{16a_i^2}b_{3,4} - \frac{a^2}{8a_i^3}b_{3,5} - \frac{3a}{16a_i^2}b_{3,6} - \frac{3.9}{2.4.4}\frac{a^2}{a_i^3}b_{5,3} + \frac{3.3}{2.4}\frac{a^3}{a_i^4}b_{5,4}$$

$$-\frac{3a^2}{2.4.2}\frac{(2a^2 - 3a_i^2)}{a_i^5}b_{5,5} - \frac{3}{2.4}\frac{a^3}{a_i^4}b_{5,6} - \frac{3a^2}{2.4.4a_i^3}b_{5,7}$$

changing b_3 into $-\frac{3}{4}b_5$, and b_5 into $-\frac{5}{6}b_7$, we have

$$\frac{3a}{64a_i^2}b_{5,4} + \frac{3a^2}{32a_i^3}b_{5,5} + \frac{9}{64}\frac{a}{a_i^2}b_{5,6} + \frac{3.9.5}{2.4.4.6}\frac{a^2}{a_i^3}b_{7,3} - \frac{3.3.5}{2.4.6}\frac{a^3}{a_i^4}b_{7,4}$$

$$+\frac{3.5a^2}{2.4.2}\frac{(2a^2 - 3a_i^2)}{a^5}b_{7,5} + \frac{3.5}{2.4.6}\frac{a^3}{a_i^4}b_{7,6} + \frac{3.5}{2.4.4.6}\frac{a^2}{a_i^3}b_{7,7}$$

$$= \frac{3}{64}\frac{a}{a_i^2}b_{5,4} + \frac{3}{32}\frac{a^2}{a_i^3}b_{5,5} + \frac{9}{64}\frac{a}{a_i^2}b_{5,6} + \frac{3.5}{8.6}\frac{a^2}{a_i^3}\left\{\frac{a^2 + a_i^2}{a_i^2}b_{7,5} - \frac{a}{a_i}b_{7,4} - \frac{a}{a_i}b_{7,6}\right\}$$

$$+\frac{3.9.5}{8.4.6}\frac{a^2}{a_i^3}\left\{b_{7,3} - b_{7,5}\right\} - \frac{3.5}{4.6}\frac{a^3}{a_i^4}\left\{b_{7,4} - b_{7,6}\right\} - \frac{3.5}{32.6}\frac{a^2}{a_i^3}\left\{b_{7,5} - b_{7,7}\right\}$$

and since $b_{5,5} = \frac{a^2 + a_i^2}{a_i^3}b_{7,5} - \frac{a}{a_i}b_{7,4} - \frac{a}{a_i}b_{7,6}$

$$4\,b_{5,4} = \frac{5}{2}\frac{a}{a_i}\left\{b_{7,3} - b_{7,5}\right\} \quad 5\,b_{5,5} = \frac{5}{2}\frac{a}{a_i}\left\{b_{7,4} - b_{7,6}\right\} \quad 6\,b_{5,5} = \frac{5}{2}\frac{a}{a_i}\left\{b_{7,5} - b_{7,7}\right\}$$

$$= \frac{3}{64}\frac{a}{a_i^2}b_{5,4} + \frac{3}{32}\frac{a^2}{a_i^3}b_{5,5} + \frac{9}{64}\frac{a}{a_i^2}b_{5,6} + \frac{15}{48}\frac{a^2}{a_i^3}b_{5,5} + \frac{27}{24}\frac{a}{a_i^2}b_{5,4} - \frac{15}{12}\frac{a^2}{a_i^3}b_{5,5} - \frac{3}{16}\frac{a}{a_i^2}b_{5,6}$$

$$= \frac{75}{64}\frac{a}{a_i^2}b_{5,4} - \frac{27}{32}\frac{a^2}{a_i^3}b_{5,5} - \frac{3}{64}\frac{a}{a_i^2}b_{5,6}$$

$$R_{54} = -\frac{a}{16a_i^2}b_{3,3} - \frac{a^2}{8a_i^3}b_{3,4} - \frac{3a}{16a_i^2}b_{3,5} - \frac{3.9a^2}{2.4.4a_i^3}b_{5,2} + \frac{3.3a^3}{2.4a_i^4}b_{5,3}$$

$$-\frac{3a^2(2a^2 - 3a_i^2)}{2.4.2a_i^5}b_{5,4} - \frac{3a^3}{2.4a_i^4}b_{5,6} - \frac{3a^2}{2.4.4a_i^3}b_{5,6}\,.$$

Similar changes and reductions give

$$\frac{57a}{64a_i^2}b_{5,3} - \frac{19a^2}{32a_i^3}b_{5,4} - \frac{a}{64a_i^2}b_{5,5}$$

$$R_{155} = \frac{3}{32}\frac{a}{a_{,}}\left\{\frac{3}{4}\frac{a}{a_{,}^2}b_{5,3} - \frac{a^2}{2a_{,}^3}b_{5,4} - \frac{a}{4a_{,}^2}b_{5,5}\right\} + \frac{3}{16}\frac{a^2}{a_{,}^2}\left\{\frac{3}{4}\frac{a}{a_{,}^2}b_{5,4} - \frac{a^2}{2a_{,}^3}b_{5,5} - \frac{a}{4a_{,}^2}b_{5,6}\right\}$$

$$+ \frac{9a}{32a_{,}}\left\{\frac{3a}{4a_{,}^2}b_{5,5} - \frac{a^2}{2a_{,}^3}b_{5,6} - \frac{a}{4a_{,}^2}b_{5,7}\right\} - \frac{a^2}{a_{,}^2}\left\{\frac{75}{64}\frac{a}{a_{,}^2}b_{5,4} - \frac{27}{32}\frac{a^2}{a_{,}^3}b_{5,5} - \frac{3}{64}\frac{a}{a_{,}^2}b_{5,6}\right\}$$

$$+ \frac{3a}{2a_{,}}\left\{\frac{57}{64}\frac{a}{a_{,}^2}b_{5,3} - \frac{19}{32}\frac{a^2}{a_{,}^3}b_{5,4} - \frac{a}{64a_{,}^2}b_{5,5}\right\}$$

and adding the terms which depend upon b_3,

$$R_{155} = \frac{a}{96a_{,}^2}b_{3,4} - \frac{a^2}{16a_{,}^3}b_{3,5} + \frac{a}{12a_{,}^2}b_{3,6} + \frac{45}{32}\frac{a^2}{a_{,}^3}b_{5,3} - \frac{63}{32}\frac{a^3}{a_{,}^4}b_{5,4} + \frac{(21a_{,}^2 + 96a^2)}{128a_{,}^5}a^2 b_{5,5}$$

$$- \frac{9}{64}\frac{a^3}{a_{,}^4}b_{5,6} - \frac{9}{128}\frac{a^2}{a_{,}^3}b_{5,7}$$

which may be still further reduced. R_{174}, R_{213}, R_{273}, and R_{312} may be obtained in a similar manner.

The following Table shows the arguments which, by their combination with the arguments 1, 2, 3, 12, 13, 31, 32, 64, 65, 73, 74, 112, and 113, by addition and subtraction produce the arguments 155, 174, 213, 273, and 312.

	1	2	3	12	13	31	32	64	65	73	74	112	113	
155	154	153	152	53	52	......	......	11	......	192	191	......	......	155
	156	157	158	......	......	......	......	......	− 10	......	......	......	......	
174	173	172	171	72	71	53	52	......	......	11	......	192	191	174
	175	176	177	......	......	......	......	− 30	− 41	......	− 10	......	......	
213	212	211	−210.	111	......	72	71	......	......	......	......	11	......	213
	214	215	216	......	−110	......	......	−231	−232	− 30	− 41	......	− 10	
273	272	271	−270.	131	......	......	......	......	......	330	......	......	......	273
	274	275	276	......	−130	......	......	−291	−292	......	−331	......	......	
312	311	−310.	−321.	......	......	131	......	......	......	......	......	330	......	312
	313	314	315	......	......	......	−130	......	......	−291	−292	......	−331	

If

$$r\,\delta.\frac{1}{r} = r'_1\cos(nt - n_{,}t) + r'_2\cos(2nt - 2n_{,}t) + r'_3\cos(3nt - 3n_{,}t) + er'_{12}\cos(nt - 2n_{,}t + \varpi)$$

$$+ er'_{13}\cos(2nt - 3n_{,}t + \varpi) + \&c.$$

$$r_{,}\,\delta.\frac{1}{r_{,}} = r_{,}'_1\cos(nt - n_{,}t) + r_{,}'_2\cos(2nt - 2n_{,}t) + r_{,}'_3\cos(3nt - 3n_{,}t) + er_{,}'_{12}\cos(nt - 2n_{,}t + \varpi)$$

$$+ er_{,}'_{13}\cos(2nt - 3n_{,}t + \varpi) + \&c.$$

$$\delta\lambda = \lambda_1\sin(nt - n_{,}t) + \lambda_2\sin(2nt - 2n_{,}t) + \lambda_3\sin(3nt - 3n_{,}t) + e\lambda_{12}\sin(nt - 2n_{,}t + \varpi)$$

$$+ e\lambda_{13}\sin(2nt - 3n_{,}t + \varpi) + \&c.$$

$$\delta\lambda_{,} = \lambda_{,1}\sin(nt-n_{,}t) + \lambda_{,2}\sin(2nt-2n_{,}t) + \lambda_{,3}\sin(3nt-3n_{,}t) + e\lambda_{,12}\sin(nt-2n_{,}t+\varpi) + e\lambda_{,13}\sin(2nt-3n_{,}t+\varpi) + \&c.$$

Supposing that the arguments 1, 2, 3, 12, 13, 31, 32, 64, 65, 73, 74, 112, 113, 155, 174, 213, 273, and 312 are alone sensible in $\delta\,.\,\frac{1}{r}$, $\delta\lambda$, $\delta\frac{1}{r_{,}}$ and $\delta\lambda_{,}$ the coefficient of $e^3\cos(2nt-5n_{,}t+3\varpi)$ in the expression for δR or δR_{155}

$$\begin{aligned}
&= -\frac{1}{2}\left\{\frac{a\,d\,.\,R_{154}}{da}+\frac{a\,d\,.\,R_{156}}{da}\right\}r'_{1} + \left\{2R_{154}-3R_{156}\right\}\left\{\lambda_1-\lambda_{,1}\right\} - \frac{1}{2}\left\{\frac{a\,d\,R_{153}}{da}+\frac{a\,d\,R_{157}}{da}\right\}r'_{2}\\
&\quad+\frac{1}{2}\left\{3R_{153}-7R_{157}\right\}\left\{\lambda_2-\lambda_{,2}\right\} - \frac{1}{2}\left\{\frac{a\,d\,.\,R_{152}}{da}+\frac{a\,d\,.\,R_{158}}{da}\right\}r'_{3}\\
&\quad+\left\{R_{152}-4R_{158}\right\}\left\{\lambda_3-\lambda_{,3}\right\} - \frac{a\,d\,.\,R_{53}}{2\,da}r'_{12} + \frac{3}{2}R_{53}\left\{\lambda_{12}-\lambda_{,12}\right\}\\
&\quad-\frac{a\,d\,.\,R_{52}}{2\,da}r'_{13} + R_{52}\left\{\lambda_{13}-\lambda_{,13}\right\} - \frac{a\,d\,.\,R_{64}}{2\,da}r'_{11} + 2R_{64}\left\{\lambda_{11}-\lambda_{,11}\right\}\\
&\quad-\frac{a\,d\,.\,R_{65}}{2\,da}r'_{10} - \frac{5}{2}R_{65}\left\{\lambda_{10}-\lambda_{,10}\right\} - \frac{a\,d\,R_{192}}{2\,da}r'_{73} - R_{192}\left\{\lambda_{73}-\lambda_{,73}\right\} - \frac{a\,d\,R_{193}}{2\,da}r'_{74}\\
&\quad-\frac{1}{2}R_{193}\left\{\lambda_{74}-\lambda_{,74}\right\} - \frac{a\,d\,.\,R_0}{da}r'_{155} - \frac{1}{2}\left\{\frac{a_{,}d\,.\,R_{154}}{da_{,}}+\frac{a_{,}d\,.\,R_{156}}{da_{,}}\right\}r'_{,1}\\
&\quad-\frac{1}{2}\left\{\frac{a_{,}d\,.\,R_{153}}{da_{,}}+\frac{a_{,}d\,.\,R_{157}}{da_{,}}\right\}r'_{,2} - \frac{1}{2}\left\{\frac{a_{,}d\,R_{152}}{da_{,}}+\frac{a_{,}d\,R_{158}}{da_{,}}\right\}r'_{,3} - \frac{a_{,}d\,.\,R_{53}}{2\,da_{,}}r'_{,12}\\
&\quad-\frac{a_{,}d\,.\,R_{52}}{2\,da_{,}}r'_{,13} - \frac{a_{,}d\,.\,R_{64}}{2\,da_{,}}r'_{,11} - \frac{a_{,}d\,.\,R_{65}}{2\,da_{,}}r'_{,10} - \frac{a_{,}d\,R_{192}}{2\,da_{,}}r_{,73} - \frac{a_{,}d\,R_{193}}{2\,da_{,}}r_{,74} - \frac{a_{,}d\,.\,R_0}{da_{,}}r'_{,155}
\end{aligned}$$

In the same way the expression for $\delta\,.\,R_{174}$, $\delta\,.\,R_{213}$, $\delta\,.\,R_{273}$, and $\delta\,.\,R_{312}$ may be found from the preceding Table.

If $a < a_{,}$ and

$$\left\{1-\frac{a}{a_{,}}\cos\theta+\frac{a^2}{a_{,}^2}\right\}^{-\frac{1}{2}} = \tfrac{1}{2}b_{1,0}{}^{*} + b_{1,1}\cos\theta + b_{1,2}\cos 2\theta \;\&c.$$

$$\left\{1-\frac{a}{a_{,}}\cos\theta+\frac{a^2}{a_{,}^2}\right\}^{-\frac{3}{2}} = \tfrac{1}{2}b_{3,0}{}^{*} + b_{3,1}\cos\theta + b_{3,2}\cos 2\theta \;\&c.$$

$$R = m_{,}\left\{\frac{a}{a_{,}^2}\left(\cos^2\frac{\iota_{,}}{2}-\frac{e^2+e_{,}^2}{2}\right\}\cos(nt-n_{,}t)\,\dagger\right.$$

$$-\frac{3m_{,}}{2}\frac{a}{a_{,}^2}e\cos(n_{,}t-\varpi) + \frac{m_{,}a}{a_{,}^2}e\cos(2nt-n_{,}t-\varpi) + \frac{2m_{,}a}{2a_{,}^2}e_{,}\cos(nt-2n_{,}t+\varpi_{,})$$

* The notation is slightly changed from that used before.

† ε and $\varepsilon_{,}$ which accompany nt and $n_{,}t$ are omitted for convenience.

General expression for the development of R.

$$+\frac{m_{\prime}a}{8a_{\prime}^2}e^2\cos(nt+n_{\prime}t-2\varpi)+\frac{3m_{\prime}a}{8a_{\prime}^2}e^2\cos(3nt-n_{\prime}t-2\varpi)-\frac{3m_{\prime}a}{a_{\prime}^2}ee_{\prime}\cos(2n_{\prime}t-\varpi-\varpi_{\prime})$$

$$+\frac{m_{\prime}a}{a_{\prime}^2}ee_{\prime}\cos(2nt-2n_{\prime}t-\varpi+\varpi_{\prime})+\frac{27}{8}\frac{m_{\prime}a}{a_{\prime}^2}e_{\prime}^2\cos(nt-3n_{\prime}t+2\varpi_{\prime})$$

$$+\frac{m_{\prime}a}{8a_{\prime}^2}e_{\prime}^2\cos(nt+n_{\prime}t-2\varpi_{\prime})+\frac{m_{\prime}a}{a_{\prime}^2}\sin^2\frac{\iota_{\prime}}{2}\cos(nt+n_{\prime}t-2\nu_{\prime})$$

$$+m_{\prime}\Sigma\left\{-\frac{b_{1,i}}{2a_{\prime}}+\frac{a}{4a_{\prime}^2}\sin^2\frac{\iota_{\prime}}{2}\left(b_{3,i-1}+b_{3,i+1}\right)\right.$$

$$\left.+\frac{a(e^2+e_{\prime}^2)}{16a_{\prime}^2}\Big((3i-1)b_{3,i-1}-(3i+1)b_{3,i+1}\Big)\right\}\cos i(nt-n_{\prime}t)$$

$$+m_{\prime}\Sigma\left\{-\frac{a}{4a_{\prime}^2}b_{3,i-1}-\frac{a^2}{2a_{\prime}^3}b_{3,i}+\frac{3a}{4a_{\prime}^2}b_{3,i+1}\right\}e\cos\Big(i(nt-n_{\prime}t)+nt-\varpi\Big)$$

$$+m_{\prime}\Sigma\left\{\frac{3}{4}\frac{a}{a_{\prime}^2}b_{3,i-1}-\frac{1}{2a_{\prime}}b_{3,i}-\frac{a}{4a_{\prime}^2}b_{3,i+1}\right\}e_{\prime}\cos\Big(i(nt-n_{\prime}t)+n_{\prime}t-\varpi_{\prime}\Big)$$

$$+m_{\prime}\Sigma\left\{-\frac{(2+i)}{16}\frac{a}{a_{\prime}^2}b_{3,i-1}-\frac{(1+i)}{2}\frac{a^2}{a_{\prime}^3}b_{3,i}\right.$$

$$\left.+\frac{(8+9i)}{16}\frac{a}{a_{\prime}^2}b_{3,i+1}\right\}e^2\cos\Big(i(nt-n_{\prime}t)+2nt-2\varpi\Big)$$

$$+m_{\prime}\Sigma\left\{\frac{(3+9i)}{8}\frac{a}{a_{\prime}^2}b_{3,i-1}-\frac{i}{a_{\prime}}b_{3,i}\right.$$

$$\left.-\frac{(1+i)}{8}\frac{a}{a_{\prime}^2}b_{3,i+1}\right\}ee_{\prime}\cos\Big(i(nt-n_{\prime}t)+nt+n_{\prime}t-\varpi-\varpi_{\prime}\Big)$$

$$+m_{\prime}\Sigma\left\{-\frac{(1+3i)}{8}\frac{a}{a_{\prime}^2}b_{3,i-1}\right.$$

$$\left.+\frac{3(1+i)}{8}\frac{a}{a_{\prime}^2}b_{3,i+1}\right\}ee_{\prime}\cos\Big(i(nt-n_{\prime}t)+nt-n_{\prime}t-\varpi+\varpi_{\prime}\Big)$$

$$+m_{\prime}\Sigma\left\{\frac{(8-9i)}{16}\frac{a}{a_{\prime}^2}b_{3,i-1}+\frac{(1-i)}{2a_{\prime}}b_{3,i}\right.$$

$$\left.-\frac{(2-i)}{16}\frac{a}{a_{\prime}^2}b_{3,i+1}\right\}e_{\prime}^2\cos\Big(i(nt-n_{\prime}t)+2n_{\prime}t-2\varpi_{\prime}\Big)$$

$$-m_{\prime}\Sigma\frac{a}{2a_{\prime}^2}b_{3,i-1}\sin^2\frac{\iota_{\prime}}{2}\cos\Big(i(nt-n_{\prime}t)+2n_{\prime}t-2\nu_{\prime}\Big)$$

i being every whole number, positive and negative and zero, and observing that $b_{m,n}=b_{m,-n}$. Considering only the terms multiplied by e and $e_{\prime}$,

$$r\left(\frac{dR}{dr}\right)=-\frac{3m_{\prime}}{2}\frac{a}{a_{\prime}^2}e\cos(n_{\prime}t-\varpi)+\frac{m_{\prime}a}{2a_{\prime}^2}e\cos(2nt-n_{\prime}t-\varpi)$$

$$+ \frac{m_{\prime} a}{2 a_{\prime}^2} e_{\prime} \cos (n t - 2 n_{\prime} t + \varpi_{\prime})$$

$$+ m_{\prime} \Sigma \left\{ - \frac{i}{4} \frac{a}{a_{\prime}^2} b_{3,i-1} + \frac{(1+2i)}{2} \frac{a^2}{a_{\prime}^3} b_{3,i} - \frac{3i}{4} \frac{a}{a_{\prime}^2} b_{3,i+1} \right\} e \cos \Big(i (n t - n_{\prime} t) + n t - \varpi \Big)$$

$$+ m_{\prime} \Sigma \left\{ - \frac{3(1+i)}{4} \frac{a}{a_{\prime}^2} b_{3,i-1} + \frac{i a}{a_{\prime}} b_{3,i} + \frac{(1-i)}{4} b_{3,i+1} \right\} e_{\prime} \cos \Big(i (n t - n_{\prime} t) + n_{\prime} t - \varpi_{\prime} \Big)$$

$$\frac{\mathrm{a}}{r} = - \frac{m_{\prime}}{\mu} \frac{n^2}{(3n - n_{\prime})(n - n_{\prime})} \left\{ \frac{2n}{2n - n_{\prime}} + \frac{1}{2} \right\} \frac{a^2}{a_{\prime}^2} e \cos (2 n t - n_{\prime} t - \varpi)$$

$$- \frac{m_{\prime}}{\mu} \frac{3 n^2}{2(n - n_{\prime})(n + n_{\prime})} \frac{a^2}{a_{\prime}^2} e \cos (n_{\prime} t - \varpi)$$

$$+ \frac{m_{\prime}}{\mu} \frac{n^2}{n_{\prime}(2n - 2n_{\prime})} \left\{ \frac{2n}{(n - 2n_{\prime})} + 1 \right\} \frac{a^2}{a_{\prime}^2} e_{\prime} \cos (n t - 2 n_{\prime} t + \varpi_{\prime})$$

$$+ \Sigma \frac{n^2}{\Big(i(n - n_{\prime}) + 2n \Big) i (n - n_{\prime})} \left[\frac{3 \Big(i(n - n_{\prime}) + n \Big)}{2 n^2} 2 r_i{}^{*} \right.$$

$$- \frac{m_{\prime}}{\mu} \left\{ \frac{2(1+i)n}{i(n - n_{\prime}) + n} \left\{ - \frac{a^2}{4 a_{\prime}^2} b_{3,i-1} - \frac{a^3}{2 a_{\prime}^3} b_{3,i} + \frac{3 a^2}{4 a_{\prime}^2} b_{3,i+1} \right\} \right.$$

$$\left. \left. - \frac{i}{4} \frac{a^2}{a_{\prime}^2} b_{3,i-1} + \frac{(1+2i)}{2} \frac{a^3}{a_{\prime}^3} b_{3,i} - \frac{3i}{4} \frac{a^2}{a_{\prime}^2} b_{3,i+1} \right\} \right] e \cos \Big(i (n t - n_{\prime} t) + n t - \varpi \Big)$$

$$+ \frac{m_{\prime}}{\mu} \Sigma \frac{n^2}{(1-i)(n - n_{\prime}) \Big((i+1)(n - n_{\prime}) + 2 n_{\prime} \Big)} \left\{ \frac{2 i n}{i(n - n_{\prime}) + n_{\prime}} \left\{ \frac{3 a^2}{4 a_{\prime}^2} b_{3,i-1} \right. \right.$$

$$\left. - \frac{a}{2 a_{\prime}} b_{3,i} - \frac{a^2}{4 a_{\prime}^2} b_{3,i+1} \right\} - \frac{3(1+i)}{4} \frac{a^2}{a_{\prime}^2} b_{3,i-1}$$

$$\left. + \frac{i a}{a_{\prime}} b_{3,i} + \frac{(1-i)}{4} \frac{a^2}{a_{\prime}^2} b_{3,i+1} \right\} e_{\prime} \cos \Big(i (n t - n_{\prime} t) + n_{\prime} t - \varpi_{\prime} \Big)$$

$$\lambda = - \left\{ \frac{3 n^2}{2 n_{\prime}^2} + \frac{n^2}{n_{\prime}(n - n_{\prime})} \frac{m_{\prime}}{\mu} \right\} \frac{a^2}{a_{\prime}^2} e \sin (n_{\prime} t - \varpi)$$

$$- \left\{ \frac{n^2}{(2n - n_{\prime})^2} + \frac{n^2}{(2n - n_{\prime})(n - n_{\prime})} \right\} \frac{m_{\prime}}{\mu} \frac{a^2}{a_{\prime}^2} e \sin (2 n t - n_{\prime} t - \varpi)$$

$$- \frac{2 n^2}{(n - 2 n_{\prime})^2} \frac{m_{\prime}}{\mu} \frac{a^2}{a_{\prime}^2} e_{\prime} \sin (n t - 2 n_{\prime} t + \varpi_{\prime})$$

* r_i being the coefficient of $\cos \Big(i (n t - n_{\prime} t) \Big)$ in the expression for $\frac{\mathrm{a}}{r}$.

$$+ \Sigma \dagger \frac{n}{i(n-n_{\prime})+n} \left\{ 2\left(r^* + \frac{r_i}{2}\right) - \frac{m_{\prime} n i}{\mu \left(i(n-n_{\prime})+n\right)} \left(-\frac{a^2}{4a_{\prime}^2} b_{3,i-1} - \frac{a^3}{2a_{\prime}^3} b_{3,i} \right.\right.$$

$$\left.\left. + \frac{3a^2}{4a_{\prime}^2} b_{3,i+1} \right) + \frac{m_{\prime} n}{\mu(n-n_{\prime})} \frac{a}{a_{\prime}} b_{1,i} \right\} e \sin \left(i(nt - n_{\prime} t) + nt - \varpi \right)$$

$$+ \Sigma \frac{n}{i(n-n_{\prime})+n_{\prime}} \left\{ 2r^* - \frac{m_{\prime} n i}{\mu \left(i(n-n_{\prime})+n_{\prime}\right)} \left(\frac{3}{4} \frac{a^2}{a_{\prime}^2} b_{3,i-1} - \frac{a}{2a_{\prime}} b_{3,i} \right.\right.$$

$$\left.\left. - \frac{a^2}{4a_{\prime}^2} b_{3,i+1} \right) e_{\prime} \sin \left(i(nt - n_{\prime} t) + n_{\prime} t - \varpi_{\prime} \right) \right.$$

If $a > a_{\prime}$, and

$$\left\{1 - \frac{a_{\prime}}{a} \cos\theta + \frac{a_{\prime}^2}{a^2}\right\}^{-\frac{1}{2}} = \tfrac{1}{2} b_{1,0} + b_{1,1} \cos\theta + b_{1,2} \cos 2\theta + \&c.$$

$$\left\{1 - \frac{a_{\prime}}{a} \cos\theta + \frac{a_{\prime}^2}{a^2}\right\}^{-\frac{3}{2}} = \tfrac{1}{2} b_{3,0} + b_{3,1} \cos\theta + b_{3,2} \cos 2\theta + \&c.$$

the value of R may be easily inferred from the value which it has in the former case. Considering only the terms multiplied by the eccentricities

$$r\left(\frac{dR}{dr}\right) = -\frac{3m_{\prime}}{2} \frac{a}{a_{\prime}^2} e \cos(nt - \varpi) + \frac{m_{\prime}}{2} \frac{a}{a_{\prime}^2} e \cos(2nt - n_{\prime} t - \varpi)$$

$$+ \frac{m_{\prime}}{2} \frac{a}{a_{\prime}^2} e_{\prime} \cos(nt - 2n_{\prime} t + \varpi_{\prime})$$

$$+ m_{\prime} \Sigma \left\{ -\frac{i}{4} \frac{a_{\prime}}{a^2} b_{3,i-1} + \frac{(1+2i)}{2a} b_{3,i} \right.$$

$$\left. - \frac{3i}{4} \frac{a_{\prime}}{a^2} b_{3,i+1} \right\} e \cos\left(i(nt - n_{\prime} t) + nt - \varpi \right)$$

$$+ m_{\prime} \Sigma \left\{ -\frac{3(1+i)}{4} \frac{a_{\prime}}{a^2} b_{3,i-1} + \frac{i a_{\prime}^2}{a^3} b_{3,i} \right.$$

$$\left. + \frac{(1-i)}{4} \frac{a_{\prime}}{a^2} b_{3,i+1} \right\} e_{\prime} \cos\left(i(nt - n_{\prime} t) + n_{\prime} t - \varpi_{\prime} \right)$$

All these expressions are to a certain extent arbitrary, on account of the equation which connects $b_{3,i-1}$, $b_{3,i}$, and $b_{3,i+1}$

$$\frac{(2i+1)}{2} \frac{a}{a_{\prime}} b_{3,i+1} = \frac{i(a^2 + a_{\prime}^2)}{a_{\prime}^2} b_{3,i} - \frac{(2i-1)}{2} \frac{a}{a_{\prime}} b_{3,i-1}$$

† r^* being the coefficient of the cosine of the same argument in the expression for $\frac{a}{r}$ and excluding the case of $i = 0$.

The reader is requested to make the following corrections.

Page 50, line 4, read $q_6 = -\frac{3\,a}{2\,a_{\prime}^2} + \frac{3}{2}\,\frac{a}{a_{\prime}^2}\,b_{3,0} - \frac{a^2}{2\,a_{\prime}^3}\,b_{3,1} + \frac{a}{4\,a_{\prime}^2}\,b_{3,2}$

Page 53, line 3, read $= \frac{m_{\prime}}{\mu}\left\{\frac{2\,a^3}{a_{\prime}^3}\,b_{3,0} - \frac{5}{4}\,\frac{a^2}{a_{\prime}^2}\,b_{3,1}\right\}$

Page 247, line 1, read $\lambda = n\,t$

$$\begin{aligned} &+ \lambda_1 \sin 2\,t \\ &+ e\,\lambda_2 \sin x \\ &+ e\,\lambda_3 \sin (2\,t - x) \\ &+ e\,\lambda_4 \sin (2\,t + x) \\ &+ e_{\prime}\,\lambda_5 \sin z \quad \text{\&c. \&c.} \end{aligned}$$

for $\lambda = n\,t$

$$\begin{aligned} &+ \lambda_1 \cos 2\,t \\ &+ e\,\lambda_2 \cos x \quad \text{\&c. \&c.} \end{aligned}$$

Page 254, line 1, read $-\frac{3}{2}\,e^2\,e_{\prime} \cos (2\,t + 2\,x + z)$

[25] [30]

Page 260, line 6, read $+\left\{3 - \frac{15}{2}\right\}\,e\,e_{\prime} \cos (x - z - 2\,y)$

[89]

Page 262, line 6, read $-\frac{15}{32}\,e\,e_{\prime}^3 \cos (2\,t + x - 3\,z)$

[58]

Page 265, line 1, read $+\frac{25}{64}\,\frac{a^2}{a_{\prime}^3}\,e^3\,e_{\prime} \cos (2\,t + 3\,x + z) + \frac{3}{32}\,\frac{a^2}{a_{\prime}^3}\,e^3\,e_{\prime} \cos (3\,x - z)$

[43] [44]

Page 274, line 6, read $+\left\{2\,r_3 + r_1 - \left\{\frac{9}{2\,(2 - m - c)}\right.\right.$ &c.

Page 274, line 7, read $+\left\{2\,r_4 + r_1 - \left\{-\frac{3}{2\,(2 - m + c)}\right.\right.$ &c.

Page 291, line 9, read $+\frac{3}{16}\,\frac{a}{a_{\prime}^2}\,e_{\prime}^2 \cos (t + 2\,z)$

Page 294, line 20, read $+\frac{m_{\prime}\,a}{2\,a_{\prime}^2} \cos (2\,n\,t - n_{\prime}\,t - \varpi) + \frac{2\,m_{\prime}\,a}{a_{\prime}^2}\,e_{\prime} \cos (n\,t - 2\,n_{\prime}\,t + \varpi_{\prime})$